GRISPE

Guidelines and **R**ecommendations for **I**ntegrating **S**pecific **P**rofiled steel sheets in the **E**urocodes
(**GRISPE**)

Working Package 2

Curved Profiles

Test analysis and interpretation

30 june 2016

Deliverable D 2.4

Guidelines and Recommendations for Integrating Specific Profiled Steels sheets in the Eurocodes (GRISPE)

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1. Introduction

The subject of the tests are trapezoidal and sinusoidal sheets, which are curved by bending or rollforming during manufacturing. The influence of this manufacturing procedure on the load bearing capacity, in particular on the bending moment capacity, in comparison with the straight profile is investigated. Furthermore the bearing behavior under combined bending moment and axial compression, as it happens in curved sheets which work as an arch, is verified by some tests.

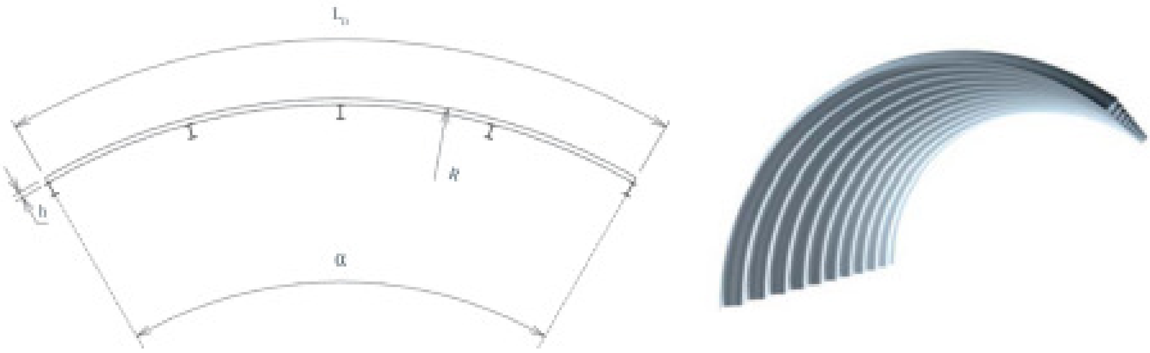

Fig. 1: Curved profile by roll forming

2. Description of the considered profiles

2.1 Cross sections

Two different profiles were tested.

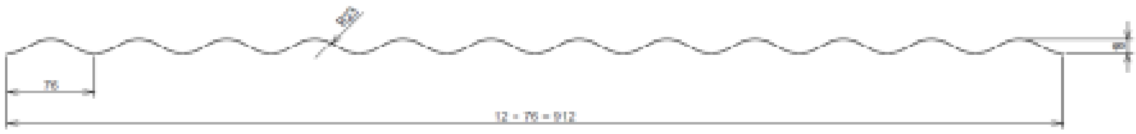

Fig. 2: Cross section of the sinusoidal profile Bacacier 18/76

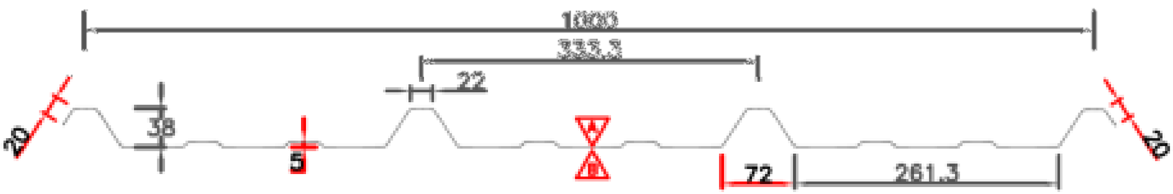

Fig. 3: Cross section of the trapezoidal sheeting Arcelor 39/333

The geometry of the used profiles was measured at 3 different specimens per batch. The results are given in [1]. The measured values are sufficiently close to the nominal values. The used specimen and the test results can be considered as representative for the nominal cross sections.

2.2 Material

The tested profiles were produced from coils steel grade S320 GD according to EN 10346. From different test specimen material samples were taken and tensile tests executed. The results are given in table 1.

profile/batch	material	nominal values t_N (mm) f_{yb} (N/mm²) f_u (N/mm²)	test no.	measured values $t_{cor,obs}$ mm	 $f_{yb,obs}$ N/mm²	 $f_{u,obs}$ N/mm²	 $A_{L=80}$ %
Bacacier 18/76	steel S320 GD	0,63 320 390	1 2 3	0,53 0,53 0,52	330 329 329	456 457 456	26,2 26,0 25,4
			mean values	0,527	329,3	456,3	25,9
Bacacier 18/76	steel S320 GD	1,00 320 390	1 2 3	0,99 1,00 0,99	342 346 358	387 387 392	29,3 27,6 27,9
			mean values	0,993	348,7	388,7	28,3
Arcelor 39/333	steel S320 GD	0,63 320 390	1 2 3	0,58 0,58 0,58	406 411 408	430 430 431	27,6 26,4 27,0
			mean values	0,580	408,3	430,3	27,0
Arcelor 39/333	steel S320 GD	1,00 320 390	1 2 3	0,96 0,96 0,95	379 384 382	425 427 426	24,7 24,5 25,4
			mean values	0,957	381,7	426,0	24,9

Table 1: Observed material properties and reference values

The scattering of the individual values among the samples of the same batch is very small; the mean values of the batch can be considered as representative for all test specimen of the same batch.

2.3 System geometry of the test specimen

Due to tolerances in the curving process the real radius of curvature differed from the designed value. For each test specimen, the real height of the arch at both longitudinal edges was measured. The mean value of the measured heights defines the real radius of curvature. Since the scattering of the heights within the test family is small, the mean radius of curvature is considered as representative for all specimen of the same family.

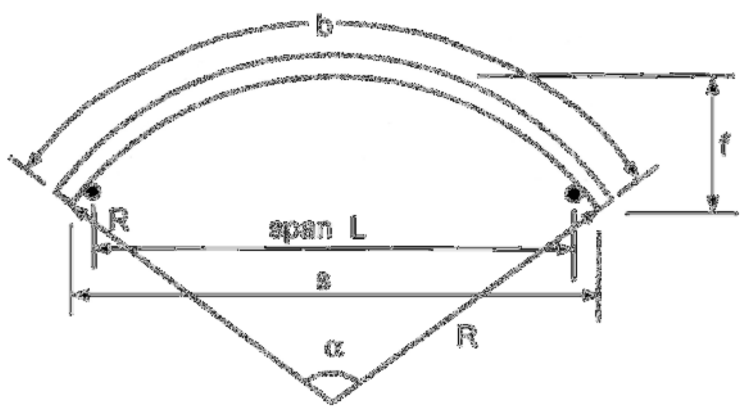

Fig. 4: Definition of the parameters of the curved profiles

test no. SSP-…	Span L (m)	height of arch (mm)			Radius (m) mean value of the family	slope at support $\alpha/2$ (arc)
		left side	right side	mean value		
18-30-063-1 18-30-063-2	2,00	40 40	47 48	43,8	11,5	0,087
18-61-063-1 18-61-063-2	2,00	45 48	56 60	52,3	9,6	0,104
18-154-063-1 18-154-063-2	2,00	120 120	115 120	118,8	4,3	0,236
18-64-100-1 18-64-100-2 18-64-100-3 18-64-100-4	3,00	75 70 64 60	60 70 60 65	65,5	17,2	0,087
18-129-100-1 18-129-100-2 18-129-100-3	3,00	110 110 90	120 125 85	106,7	10,6	0,142
18-334-100-1 18-334-100-2 18-334-100-3	3,00	320 325 315	300 295 300	309,2	3,8	0,407
39-64-063-1 39-64-063-2	3,00	37 34	34 32	34,3	32,9	0,046
39-129-063-1 39-129-063-2	3,00	116 116	116 120	117,0	9,7	0,156
39-217-063-1 39-217-063-2 39-217-063-3	3,00	205 205 210	200 205 210	205,8	5,6	0,273
39-111-100-1 39-111-100-2	4,00	74 74	82 80	77,5	25,8	0,077
39-223-100-1 39-223-100-2	4,00	190 190	190 190	190,0	10,6	0,189
39-380-100-1 39-380-100-2	4,00	320 325	327 315	321,8	6,4	0,319
H-39-217-063-1 H-39-217-063-2	3,00	200 205	210 210	206,3	5,56	0,273
H-39-380-063-1 H-39-380-063-2 H-39-380-063-3	4,00	330 340 335	350 350 345	341,7	6,02	0,338
H-39-576-063-1 H-39-576-063-2 H-39-576-063-3	5,00	460 450 460	465 455 465	459,2	7,04	0,363

Table 2: Observed arche geometry

The nominal value of the arch's height is the second number in the test's designation: 30 mm; 61 mm; 154 mm etc. In general, the realized radius of curvature was greater than designed.

The leading character "H" in the test's designation indicates the tests with horizontal support (arch tests).

3. Principles of test evaluation

3.1 Adjustment of test results

Considering the aim of the tests, the test results are not adjusted to nominal material properties (yield stress, core thickness).

With the bending tests without horizontal support, the influence of the radius of curvature on the bending moment capacity is studied. The relation between different test families is interesting, not the explicit characteristic bending moment for each individual family. A identical adjustment of all tests results of the same batch doesn’t change the relation between different families and is therefore not necessary.

The tests with arch effect are done to verify a design model by calculation. The calculation to which the test value refers is executed with the observed material properties of the test specimen. So, the calculation corresponds to the test result without any adjustment of the test result.

3.2 Characteristic values

The characteristic values of the searched bearing properties are determined by a statistical evaluation of the test results.

A test series in this context includes all tests with the same test setup and the same failure mode. So, all single span tests without horizontal support perform one test series as well as all single span tests with horizontal support (arch tests). In the first case, failure occurs by bending, in the second case by a combination of global buckling and bending.

Each test series consist of several subsets; a subset is a small series of tests with identical conditions (same profile type, same nominal sheet thickness, same test setup etc.). Normally, a subset consists of 2 or 3 identical tests.

The test results of a subset are referred to its specific mean value R_m; the statistical evaluation is done with these normalized values.

The characteristic value is

$$R_k = R_m \cdot (1 - k \cdot s)$$

R_m mean value of the subset

s standard deviation

k coefficient depending of the number of tests according to table 3

n	3	4	5	6	8	10	20	30	∞
k	-	2,63	2,33	2,18	2,00	1,92	1,76	1,73	1,64

Table 3: fractile coefficients k according to EN 1993-1.3 table A.2

4. Test evaluation of the single span tests without horizontal support

4.1 Self-weight of the test specimens

The self-weight of the test specimens is taken from the producer's brochure.

profile	thickness t (mm)	self-weight (kN/m²)
Bacacier 18/76	0,63	0,059
	1,00	0,093
Arcelor 39/333	0,63	0,060
	1,00	0,095

Table 4: self-weight of the tested profiles

4.2 Single span tests, bending moment capacity of the curved profiles

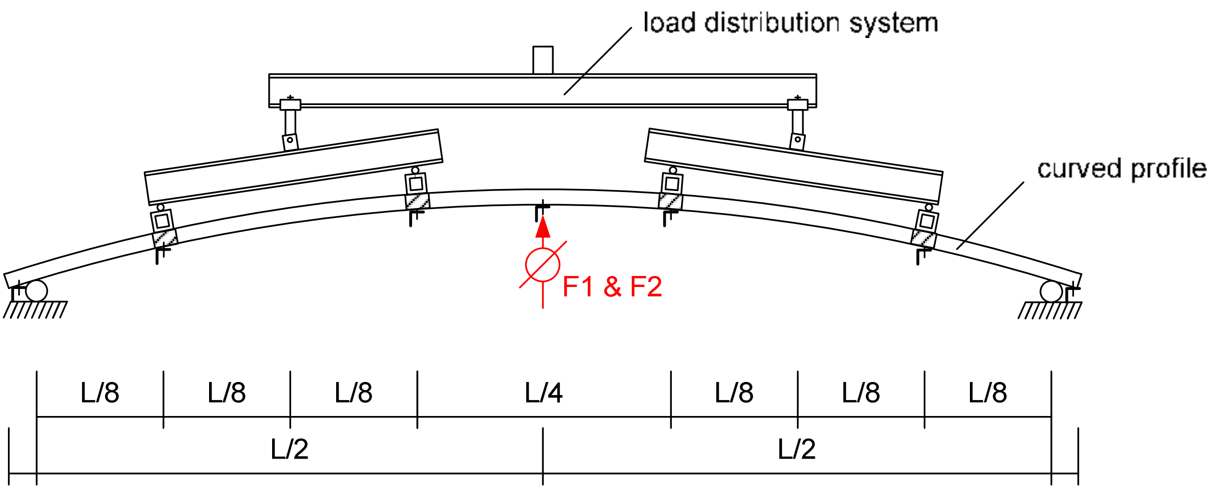

Fig. 5: Test setup single span tests

The load is applied as 4 line loads at 0,125 L – 0,25 L – 0,25 L – 0,25 L – 0,125 L. Due to the isostatic load distribution system, all 4 line loads are equal.

Since there is no horizontal support, the load creates bending moments and shear forces in the profile. Axial forces are negligible.

Fig. 6: Real setup of the single span tests without horizontal support

Maximum bending moment in span:

$$M_{c,Rk,F} = F_{u,k} / b_V * L/8 + g * L_V * [2 L – L_V] / 8$$

$M_{c,Rk,F}$ characteristic bending moment in span (kNm/m)

$F_{u,k}$ characteristic load in kN (including preload)

b_V width of the test specimen (here: b_V = 0,912 or 1,00 m)

L_V length of the test specimen (here: L_V = 2,20 or 3,20 or 4,20 m)

L span length (here: L = 2,00 or 3,00 or 4,00 m)

g self weight of the test specimen according to table 4

The detailed test evaluation is presented in the annex page 1. The main results are:

Bacacier 18/76-0,63			Bacacier 18/76-1,00			Arcelor 39/333-0,63			Arcelor 39/333-1,00		
R m	1/R 1/m	$M_{c,Rk,F}$ kNm/m	R m	1/R 1/m	$M_{c,Rk,F}$ kNm/m	R m	1/R 1/m	$M_{c,Rk,F}$ kNm/m	R m	1/R 1/m	$M_{c,Rk,F}$ kNm/m
flat	0,000	1,057	flat	0	1,727	flat	0	0,785	flat	0	1,539
11,5	0,087	1,071	17,2	0,058	1,736	32,9	0,030	0,767	25,8	0,039	1,513
9,6	0,104	1,100	10,6	0,094	1,674	9,7	0,103	0,733	10,6	0,094	1,544
4,3	0,234	1,327	3,8	0,264	1,661	5,6	0,180	0,647	6,4	0,157	1,554

Table 5: Test results: characteristic bending moment depending of the curvature 1/R

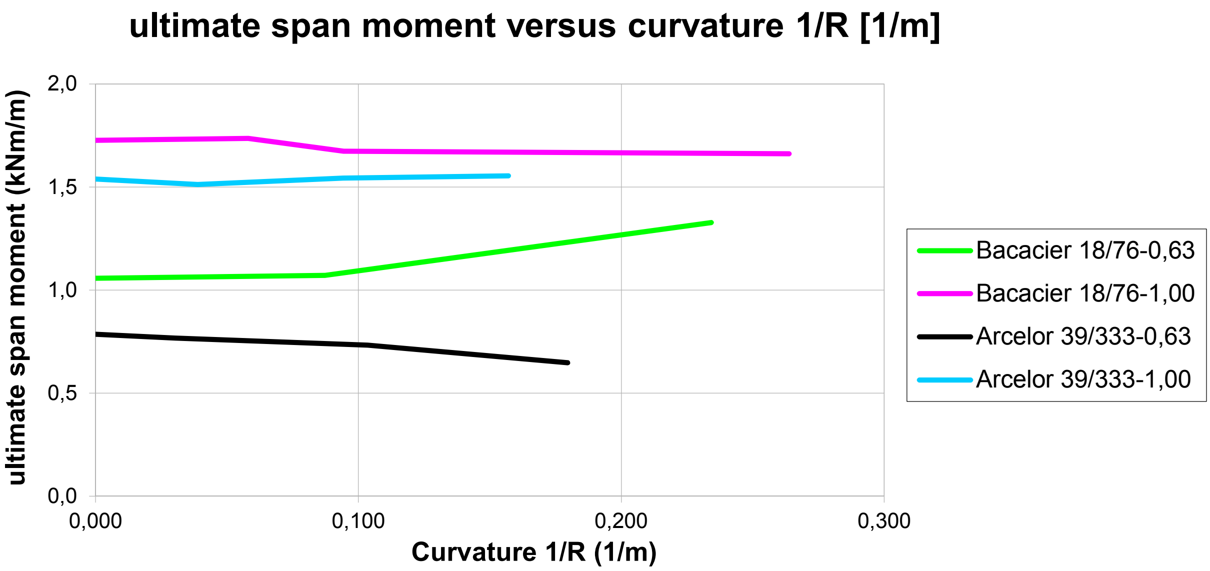

Fig. 7: Ultimate span moment versus curvature 1/R [1/m]

4.3 Design proposition for curved profiles

The curving process by bending or by rollforming creates plastic deformations of the cross section in the extreme fibres of the cross section. This leads to internal stresses in the cross section which can influence the bending moment capacity of the cross section. But the test results show, that the influence is rather small and furthermore not uniform:
For the profiles with thickness 1,0 mm, the curvature doesn't change the bending moment capacity. For the profiles with thickness 0,63 mm, the bending moment capacity is affected in both senses:

+ 25 % for the sinusoidal profile 18/76

- 15 % for the trapezoidal profile 39/333

With respect to this indifferent behaviour and regarding the low sensitivity of the bending moment capacity it is proposed to reduce the bending moment capacity by 10 % compared to the bending moment capacity of the flat profile. This reduction factor is an additional safety factor to cover the indifferent scattering; it is not a mechanically based coefficient.

$$\mathbf{M_{c,Rk,F}\ (curved\ profile) = 0{,}9 * M_{c,Rk,F}\ (flat\ profile)}$$

In the following table 6, the characteristic bending moment resistance $M_{c,Rk,F}$ determined by test and the proposed bending moment resistance for curved profiles are compared:

profile	nominal thickness t (mm)	radius of curvature R (m)	charact. bending moment (test value) $M_{c,Rk,F}$ (kNm/m)	design proposition 0,9 * $M_{c,Rk,F\ (flat)}$ (kNm/m)	ratio design/test
Bacacier 18/76	0,63	flat	1,057		
		11,5	1,071	0,951	0,89
		9,6	1,100	0,951	0,86
		4,3	1,327	0,951	0,72
	1,00	flat	1,727		
		17,2	1,736	1,554	0,89
		10,6	1,674	1,554	0,93
		3,8	1,661	1,554	0,94
Arcelor 39/333	0,63	flat	0,785		
		32,9	0,767	0,707	0,92
		9,7	0,733	0,707	0,96
		5,6	0,647	0,707	1,09
	1,00	flat	1,539		
		25,8	1,513	1,385	0,92
		10,6	1,544	1,385	0,90
		6,4	1,554	1,385	0,89

Table 6: Comparison characteristic bending moment (test) and design proposition

Other properties of the profile, in particular the resistance against punctual loads and the moment of inertia are not touched in a considerable way. The values of the flat profile remain valid also for curved profiles.

5. Test evaluation of the single span tests with horizontal support (arch tests)

5.1 Characteristic failure load

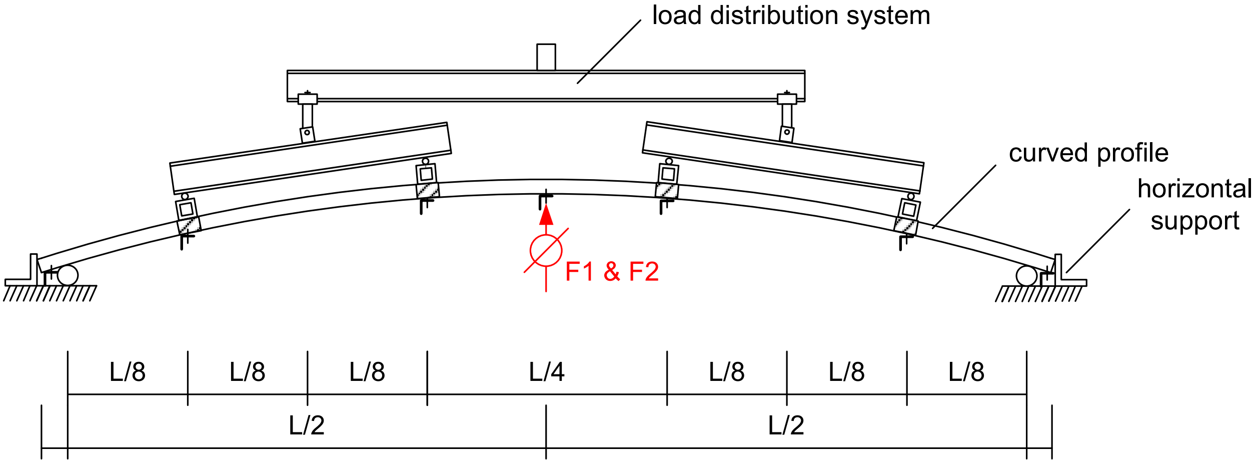

Fig. 8: Test setup of the arch tests (single span tests with horizontal support)

The load is applied as 4 line loads at 0,125 L – 0,25 L – 0,25 L – 0,25 L – 0,125 L. Due to the isostatic load distribution system, all 4 line loads are equal.

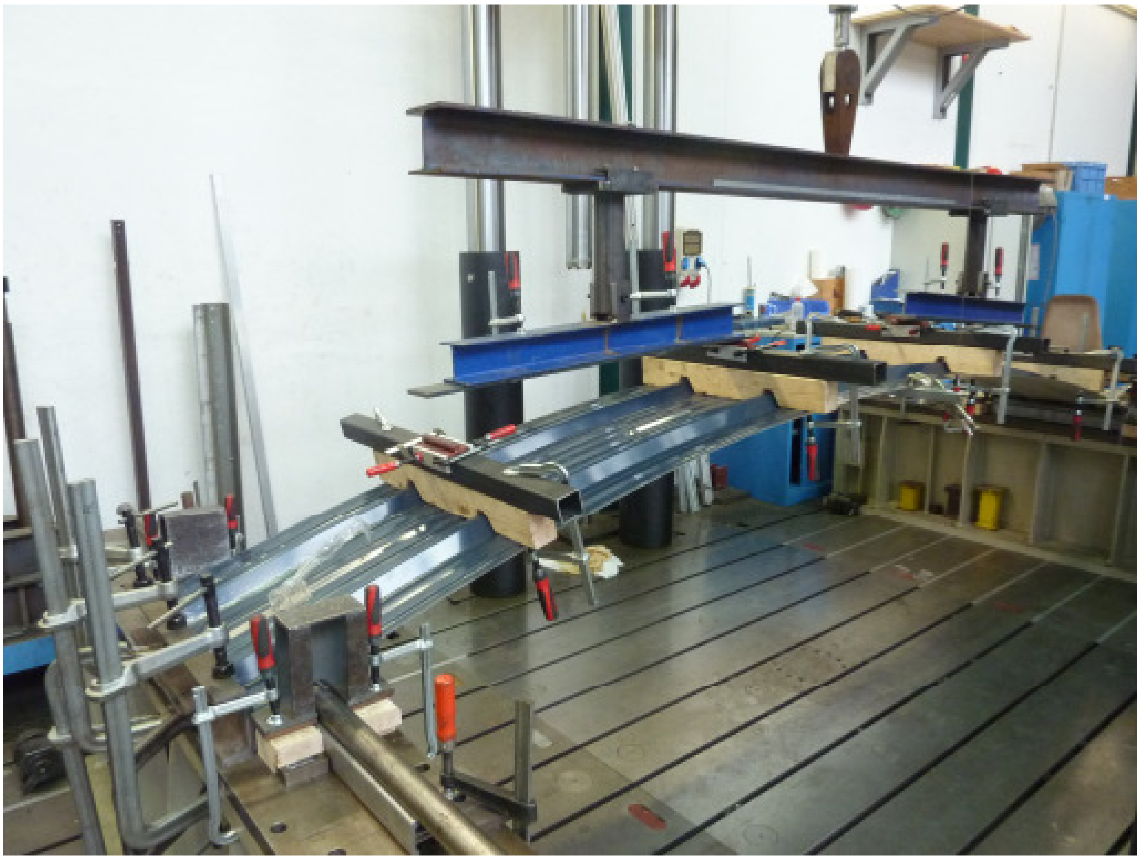

Fig. 9: Real setup of the arch tests

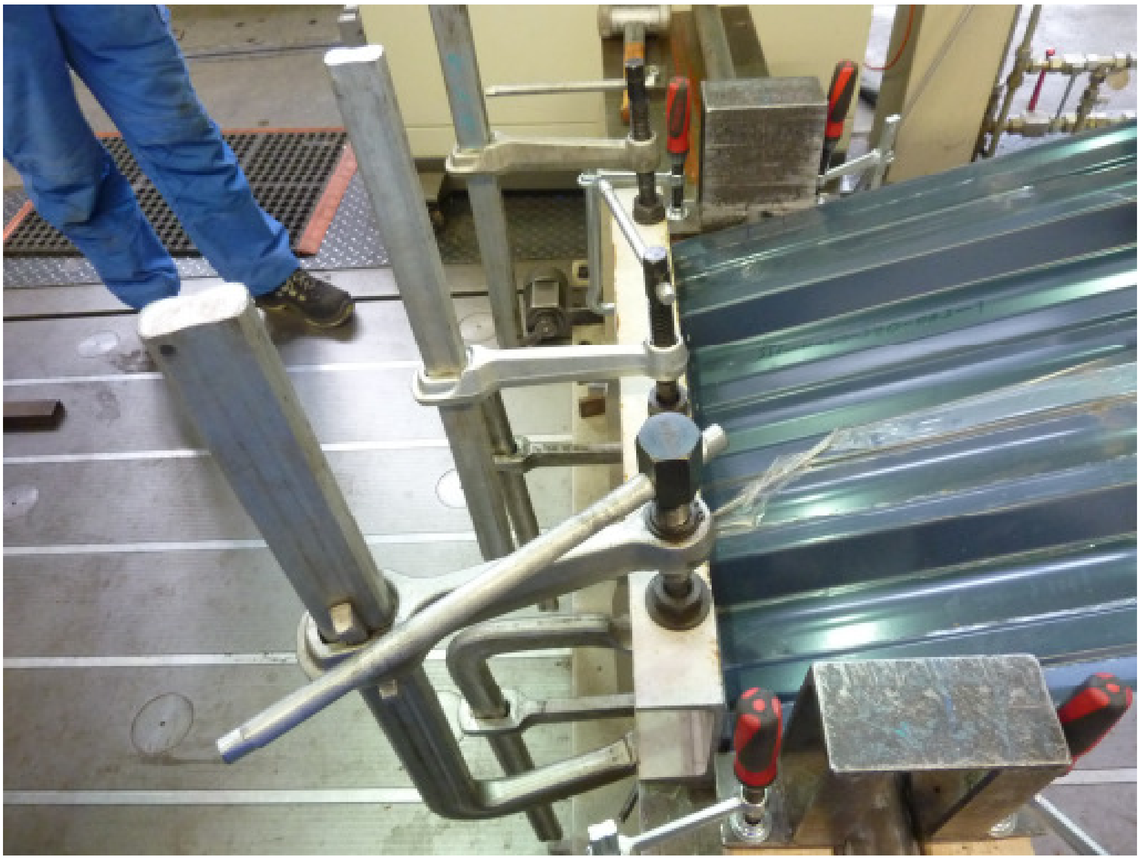

Fig. 10: Support detail of the arch tests: the clamped U-beam performs the horizontal support

The static system of the test specimen is hyperstatic; therefore the internal forces don't depend directly from the applied load, but also from the stiffness parameters of the beam and its supports. The statistic evaluation is applied on the failure loads in order to determine an individual characteristic (failure) load for each subset.

The detailed test evaluation is presented in the annex page 2. The characteristic failure loads were recalculated to the reference width of 1 m.

Profile - thickness	Radius m	span L m	char load $F_{u,k}$ kN/m
Arcelor 39/333 - 0,63	**5,56**	3,00	**11,027**
	6,02	4,00	**12,767**
	7,04	5,00	**6,615**

Table 7: Characteristic failure loads of the arch tests

5.2 Internal forces and deflections of the arches under failure load

5.2.1 General

The internal forces of the arch and its deflections under characteristic failure load are determined with the software FRILO-ESK1. The curved geometry of the arch is approached by a polygonal line. The horizontal support is performed with a spring to allow and to control horizontal deflections at the support. The self-weight of the profile is neglected. In the annex page 4 to 9, a detailed example is presented. The main results are given in chapter 5.2.4.

The internal forces and the deflections depend on the stiffness of the horizontal support, the axial stiffness EA and the bending stiffness EJ of the profile. These parameters are varied to study their influence on the internal forces. In general, as stiffer the horizontal support and as greater the axial stiffness of the profile, axial forces in the profile become greater and bending moments become smaller. In case of fixed horizontal supports (spring stiffness C = ∞) the bending moments nearly disappear, and the load is transduced to the supports almost by axial forces in the profile (arch effect). If horizontal deflections at the supports are allowed, bending moments become greater and axial forces become smaller. Furthermore, the vertical deflection at the summit becomes greater.

Variation of stiffness parameters:

- Spring stiffness of the horizontal support fixed (C = ∞)
  C_{ind} (29 … 88 kN/m/cm)
  C = 20 kN/m/cm
  C = 10 kN/m/cm

  The spring stiffness C_{ind} is different for each arch; this value is calibrated according to the condition, that the calculated vertical deflection at summit is the same as measured in the test family. Detailed calculation see chapter 5.2.2.

- Cross section properties gross cross section A_g , J_g
  effective cross section A_{ef} , J_{ef}

  Cross section values of the trapezoidal profile Arcelor 39/333 see chapter 5.2.3.

5.2.2 Calibration of the spring stiffness of the horizontal support

Since the internal forces of the arch depend on the horizontal displacement at support, it is crucial to adopt the correct spring stiffness, when the internal forces are calculated. This parameter controls the internal forces, and nearly anyone result can be found with different spring stiffnesses (cf. table 9). Considering the influence of the spring stiffness on the internal forces, the spring stiffness should be estimated “on the weak side” in order to obtain internal forces on the unfavourable side. Neglecting the horizontal displacement at support leads to internal forces which are too favourable, and consequently to an unsafe design.

In the tests, neither the horizontal support reaction nor the horizontal displacement at support were measured. So, the parameters are missing to determine the spring stiffness directly. As an alternative, the spring stiffness, which is introduced in the design model to calculate the internal forces, is chosen in a way, that the calculated vertical displacement at summit fits to the vertical displacement measured in the tests.

Calculation of internal forces is done for a system, which represents the considered subset under characteristic failure load. Each subset consists of 2 ore 3 identical tests with different individual failure loads and different individual deflection values. A common deflection value has to be developed from the tests which represent the subset. This representative value is not directly the deflection of an individual test. Hereafter, the mean deflection at characteristic failure load is considered as representative.

For each test, the vertical deflection at midspan f_{max} and the corresponding (individual) failure load F_u (test) define an individual "stiffness parameter":

Overall stiffness of the specimen $C_{f,i} = F_u / f_{max}$

The mean value of all tests of the same subset is considered as representative for this family.

$$C_f = \text{Mean}(C_{f,i})$$

Using the overall stiffness value of the family, a midspan deflection under characteristic failure load can be calculated, which is considered as representative for this family. Since the internal forces of the arch are calculated for the unit width, the result should be multiplied with the width of the test specimen.

$$f_{eq} = F_{u.k} / C_f * b_V$$

test no. SSP-H-39-…	F_u kN	deflection f_{max} at mid-span (mm)	L m	b_V m	overall stiffness specimen $C_{f,i}$ (kN/mm)	mean value stiffness C_f	repr. deflection (mm) for $F_{u,k}$, width 1 m
217-063-1	9,12	18,0	3,00	0,667	0,507	0,444	16,6
217-063-2	8,95	23,5	3,00	0,667	0,381		
380-063-1	9,49	17,6	4,00	0,667	0,539	0,589	14,4
380-063-2	11,43	19,2	4,00	0,667	0,595		
380-063-3	11,03	17,4	4,00	0,667	0,634		
576-063-1	5,67	23,6	5,00	0,667	0,240	0,329	13,4
576-063-2	5,17	14,1	5,00	0,667	0,367		
576-063-3	6,83	18,0	5,00	0,667	0,379		

Table 8: Representative midspan deflection at characteristic load level $F_{u,k}$

Calculating the arch with the Frilo-software, the spring stiffness of the horizontal support is varied and finally locked to a value, for which the calculated midspan deflection under characteristic failure load corresponds to the deflection f_{eq} according to table 8. This spring stiffness of the horizontal support is the above mentioned C_{ind}.

5.2.3 Characteristic values of the trapezoidal profile Arcelor 39/333

The characteristic values of the trapezoidal profile Arcelor 39/333 are determined according to EN 1993-1-3. The adopted material properties correspond to the values of the test specimen (mean value of the batch):

- steel core thickness t_{cor} = 0,58 mm
- yield stress $f_{y,k}$ = 408,3 N/mm²

The detailed calculation is presented in the annex page 3. The decisive values for the arch calculation and the design model are:

- section properties of the gross cross section

area	A_g =	6,58	cm²/m
moment of inertia	J_g =	9,77	cm^4/m
radius of gyration	i_g =	1,218	cm

- section properties of the effective cross section for axial compression

area	A_{ef} =	1,89	cm²/m
moment of inertia	J_{ef} =	5,21	cm^4/m
radius of gyration	i_{ef} =	1,660	cm

- load bearing values

positive bending moment	M_k^+ =	1,093	kNm/m
negative bending moment	M_k^- =	1,426	kNm/m

definition: positive bending moment = compression of the small (upper) flange
negative bending moment = tension of the small (upper) flange

5.2.4 Calculation of internal forces and deflections of the arch

The circle-shaped arch is approached by a polygon with 16 straight sections: nodes no. 1 to 17. The calculation refers to 1 m unit width of the profile. The supports at the ends (nodes 1 and 17) are equipped with springs in the horizontal direction to allow and control horizontal deflections. The supports are fixed in the vertical direction. The characteristic failure load from chapter 5.1 is applied as 4 equal vertical line loads at 0,125 L – 0,25 L – 0,25 L – 0,25 L – 0,125 L (nodes 3, 7, 11 and 13). The self-weight of the profile is neglected.

In the annex page 4 to 9, the calculation of the following configuration is presented in detail as example:

- test family H-39-380-63
- span length of the arch 4,0 m
- height of the arch 342 mm
- radius of curvature 6,02 m
- characteristic failure load 12,77 kN/m = applied load in the calculation
- gross cross section with A_g = 5,80 cm²/m and J_g = 9,94 cm^4/m
- horizontal support with spring stiffness C = 62 kN/m/cm

The main interesting results of this example are:

- At support

vertical support reaction	R_V =	6,39	kN/m
horizontal support reaction	R_H =	18,64	kN/m
horizontal displacement	f_h =	0,30	cm
max axial compression force	max N =	19,70	kN/m
chosen spring stiffness	C_{ind} =	62	kN/m/cm

- The maximum internal forces in the profile act in the load point near to the summit (node 7)

max. bending moment	max M =	0,40	kNm/m

corresponding axial compression force N_7 = 18,87 kN/m
max axial compression force max N = 19,70 kN/m

- Vertical deflection at midspan (node 9) f_V = 1,445 cm

The calculated deflection f_V corresponds to the representative deflection in the tests of this family, cf. table 8.

An overview of the main interesting results of all configurations with the variations, which are described in chapter 5.2.1, is given in table 9.

Especially the bending moments are extremely sensitive against horizontal displacements at the support. The influence is more important for small spans than for big spans.

Test setup/ span	Cross section	spring stiffness at support (kN/m/cm)	Failure load	displacements (cm)		support reactions (kN/m)		M- / N-values (kNm/m, kN/m)		
								at load point near to summit		at support
			(kN/m)	f_h (support)	f_v (summit)	R_h	R_v	max M	corresp N	max N
1 / 3,00 m	gross	fixed	11,03	0,00	0,11	20,46	5,52	0,17	20,62	21,19
		68,0	11,03	0,28	1,67	18,81	5,52	0,49	18,98	19,60
		20,0	11,03	0,79	4,55	15,76	5,52	1,08	15,94	16,65
		10,0	11,03	1,28	7,33	12,82	5,52	1,65	13,01	13,80
	effective	fixed	11,03	0,00	0,41	20,35	5,52	0,19	20,51	21,08
		88,0	11,03	0,22	1,66	19,64	5,52	0,33	19,80	20,40
		20,0	11,03	0,88	5,33	17,57	5,52	0,73	17,74	18,39
		10,0	11,03	1,55	9,08	15,46	5,52	1,14	15,64	16,35
2 / 4,00 m	gross	fixed	12,77	0,00	0,08	19,13	6,39	0,25	19,35	20,16
		62,0	12,77	0,30	1,45	18,64	6,39	0,40	18,87	19,70
		20,0	12,77	0,88	4,10	17,69	6,39	0,71	17,92	18,80
		10,0	12,77	1,64	7,56	16,45	6,39	1,11	16,69	17,62
	effective	fixed	12,77	0,00	0,39	19,09	6,39	0,26	19,31	20,13
		79,0	12,77	0,24	1,44	18,88	6,39	0,33	19,11	19,93
		20,0	12,77	0,91	4,51	18,29	6,39	0,51	18,53	19,37
		10,0	12,77	1,76	8,33	17,56	6,39	0,75	17,80	18,68
3 / 5,00 m	gross	fixed	6,62	0,00	0,02	9,23	3,31	0,16	9,36	9,81
		29,0	6,62	0,31	1,34	9,01	3,31	0,26	9,14	9,60
		20,0	6,62	0,45	1,91	8,91	3,31	0,30	9,04	9,50
		10,0	6,62	0,86	3,67	8,61	3,31	0,43	8,74	9,22
	effective	fixed	6,62	0,00	0,23	9,22	3,31	0,16	9,35	9,80
		33,0	6,62	0,28	1,33	9,12	3,31	0,21	9,09	9,70
		20,0	6,62	0,45	2,07	9,05	3,31	0,24	9,18	9,63
		10,0	6,62	0,89	3,91	8,88	3,31	0,31	9,01	9,48

green coloured lines: spring stiffness at support adapted to the midspan-deflection in test.
red marked line: example configuration considered in the detailed calculation in chapter 6.

Table 9: Internal forces and deflections of the arches under characteristic failure load

6. Design model for curved profiles with axial forces

6.1 M-N-interaction according to DIN 18807

DIN 18807 contains design rules for trapezoidal sheeting under combined bending moments and axial compression forces. It is checked, if this procedure can also be adopted for curved profiles with arch effect.

In case of compression force the following is applied:

$$\frac{N_D}{N_{dD}} \cdot \left[1 + 0{,}5 \ \cdot \alpha \left(1 - \frac{N_D}{N_{dD}}\right)\right] + \frac{M}{M_d} \leq 1$$

with
N_D design value of compressive force
M design value of bending moment
M_d design resistance of bending moment
N_{dD} design resistance of compressive force

and

slenderness ratio $\alpha = \frac{L_{cr}}{i_{ef} * \pi} \cdot \sqrt{\frac{f_{y,k}}{E}}$

with
L_{cr} buckling length
i_{ef} radius of gyration of the effective cross section

In the **M-N-interaction formula**, the coefficient α should be limited to 1 if $\alpha > 1$. But this limit is not valid, when the slenderness ratio α is used to determine the ultimate compressive stress with respect to overall buckling.

Hereafter, the DIN-procedure for combined bending moment / axial compression, adapted to curved profiles is described in detail step by step. As far as explicit calculations are presented, they refer to the test setup no. 2, span 4 m, as example (see red marked line in table 9).

- **Step 1**

Determination of the internal forces of the arch under characteristic failure load (= design load) like executed in chapter 5.2.4.
For test setup no. 2: The decisive section is node 7 = section with maximum bending moment.

M =	0,40	kNm/m
N =	18,87	kN/m

- **Step 2**

Determination of the buckling length L_{cr}
The buckling length of a circle-shaped arch can be found in the literature, for instance DIN 18 800 part 2:

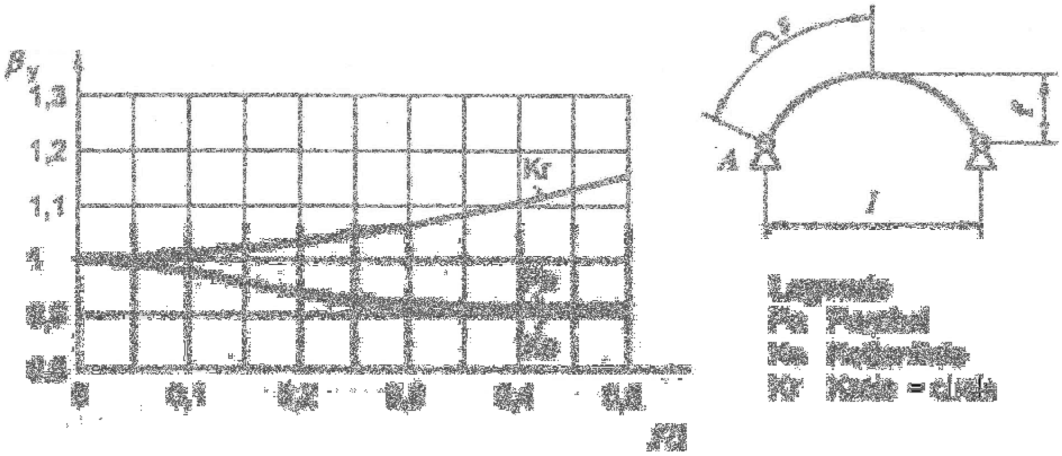

Fig. 11: buckling length coefficient b (from DIN 18800)

arch length	b	
height/span-ratio	f/L	
buckling length coefficient	β = f (height/span-ratio f/L) from diagram	
buckling Length	L_{cr} =	β * s = β * b/2

For instance test setup no. 2 H-39-380-…:

arch height	f =			342	mm
span	L =			4000	mm
slope at support	α/2 =			0,338	
radius of curvature	R =	4000 / (2 * sin 0,338)	=	6024	mm
arch length	b =	6024 * 2 * 0,338	=	4072	mm
height/span-ratio	f/L =	342 / 4000	=	0,085	
buckling length coefficient	β =			1,02	
buckling length	L_{cr} =	1,02 * 4072 / 2	=	2079	mm

- **Step 3**

Determination of the design resistance of compressive force N_{dD}

$$N_{dD} = \min(\sigma_{cd} * A_{ef} ; 0,8\ \sigma_{elg} * A_g)$$

In the calculations hereafter, the expressions max N_{dD} and ult N_{dD} are used.

Ideal buckling force $maxN_{dD} = 0,8 \cdot \sigma_{elg} \cdot A_g$

$$= 0,8 \cdot \frac{\pi^2 \cdot E \cdot J_g}{L_{cr}^2}$$

$$= 0,8 \cdot \frac{\pi^2 \cdot 21000 \cdot 9,77}{207,9^2}$$

= 37,47 kN/m

critical buckling force $ultN_{dD} = \sigma_{cd} \cdot A_{ef}$

slenderness ratio $\alpha = \frac{L_{cr}}{i_{ef} * \pi} \cdot \sqrt{\frac{f_{y,k}}{E}}$

$$\alpha = \frac{207{,}9}{1{,}66 \cdot \pi} \cdot \sqrt{\frac{408{,}3}{210000}} = 1{,}758$$

Buckling curve from DIN 18807:

α	σ_{cd}/β_S
$\alpha \leq 0{,}30$	1,00
$0{,}30 < \alpha \leq 1{,}85$	$1{,}126 - 0{,}419 \cdot \alpha$
$1{,}85 < \alpha$	$1{,}2/\alpha^2$

$$\sigma_{cd}/f_{yk} = 1{,}126 - 0{,}419 \cdot 1{,}758 = 0{,}390$$
$$\sigma_{cd} = 0{,}390 \cdot 408{,}3 = 159{,}0\ N/mm^2$$

critical buckling force $ultN_{dD} = 15{,}9 \cdot 1{,}8895 = 30{,}06\ kN/m$

decisive design resistance N_{dD} = 30,06 kN/m

- **Step 4**

Interaction bending moment / axial compression
According to DIN 18807, the slenderness value α = 1,758 should be limited to 1.

$$\frac{N_D}{N_{dD}} \cdot \left[1 + 0{,}5 \cdot \alpha \left(1 - \frac{N_D}{N_{dD}}\right)\right] + \frac{M}{M_d} \leq 1$$

$$\frac{18{,}87}{30{,}06} \cdot \left[1 + 0{,}5 \cdot 1 \left(1 - \frac{18{,}87}{30{,}06}\right)\right] + \frac{0{,}40}{1{,}093}$$

$$= 0{,}628 \cdot [1 + 0{,}5 \cdot 1\ (1 - 0{,}628)] + 0{,}366$$

$$= 0{,}745 + 0{,}366 = 1{,}11 > 1$$

In that example, the design procedure DIN 18807 leads to design on the safe side. Following the procedure with the internal ultimate forces found by test leads to an overflow of 11%. The ultimate internal forces with respect to the design limit 1 are below the failure load from test.

Beside the original DIN procedure a modified DIN procedure is checked. The modification is, that the coefficient α is not limited to 1 in the M-N-interaction formula. In that case, the step 4 is slightly different:

- **Step 4 modified DIN procedure**

Interaction bending moment / axial compression

$$\frac{N_D}{N_{dD}} \cdot \left[1 + 0{,}5 \ \cdot \alpha \left(1 - \frac{N_D}{N_{dD}}\right)\right] + \frac{M}{M_d} \leq 1$$

$$\frac{18{,}87}{30{,}06} \cdot \left[1 + 0{,}5 \cdot 1{,}758 \ \left(1 - \frac{18{,}87}{30{,}06}\right)\right] + \frac{0{,}40}{1{,}093}$$

$$= 0{,}628 \ \cdot [1 + 0{,}5 \cdot 1{,}758 \ (1 - 0{,}628)] + 0{,}366$$

$$= 0{,}833 + 0{,}366 \ = \ 1{,}20 \ > 1$$

The modified DIN procedure without limiting α is a little bit more conservative than the pure DIN approach.

Annex page 10 contains the results for all calculated configurations (test families, variation of cross section values, variation of spring stiffness).

6.2 M-N-interaction according to EN 1993-1-3

Beside of the DIN approach (chapter 6.1) the M-N-interaction with respect to overall buckling according EN 1993-1-3 is considered. It is checked, if this procedure can also be adopted for curved profiles with arch effect.

In case of compression force the following is applied

$$\left(\frac{N_D}{N_{dD}}\right)^{0{,}8} + \left(\frac{M}{M_d}\right)^{0{,}8} \leq 1$$

with

N_D design value of compressive force

M design value of bending moment

M_d design resistance of bending moment

N_{dD} design resistance of compressive force with respect to overall buckling

The compression force resistance depends on the buckling length, slenderness ratio and buckling curve. In this chapter, the buckling curves b and c are taken into account.

slenderness ratio $\bar{\lambda} = \frac{L_{cr}}{i_{ef} * \pi} \cdot \sqrt{\frac{f_{y,k}}{E}}$

The slenderness ratio $\bar{\lambda}$ is identical to the slenderness ratio α in the DIN procedure.

coefficient $\phi = 0{,}5 \cdot [1 + \alpha \, (\bar{\lambda} - 0{,}2) + \bar{\lambda}^2 \,]$

imperfection coefficient α = 0,34 for buckling curve b

α = 0,49 for buckling curve c

reduction coefficient X = $\frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}}$

The design resistance of compressive force N_{dD} is in principle identical to the above mentioned DIN approach. Only the reduction coefficient defined by the buckling curve is different.

N_{dD} = min (X * f_{yk} * A_{ef} ; 0,8 σ_{elg} * A_g)

Ideal buckling force $maxN_{dD} = 0{,}8 \cdot \sigma_{elg} \cdot A_g$

Critical buckling force $ultN_{dD} = X \cdot f_{yk} \cdot A_{ef}$

Hereafter, the EN 1993 procedure for combined bending moment / axial compression, is illustrated step by step using the same example like for DIN approach.

- **Step 1**

Determination of the internal forces of the arch under design load: like DIN procedure

M = 0,40 kNm/m
N = 18,87 kN/m

- **Step 2**

Determination of the buckling length L_{cr} = 2079 mm (like DIN procedure)

- **Step 3**

Determination of the design resistance of compressive force N_{dD}

N_{dD} = min (X * f_{yk} * A_{ef} ; 0,8 σ_{elg} * A_g)

ideal buckling force $maxN_{dD} = 0{,}8 \cdot \sigma_{elg} \cdot A_g = 37{,}47\ kN/m$ (like DIN procedure)

slenderness ratio $\bar{\lambda} = \frac{L_{cr}}{i_{ef} * \pi} \cdot \sqrt{\frac{f_{y,k}}{E}} = 1{,}758$ (like DIN procedure)

coefficient $\phi = 0{,}5 \cdot [1 + \alpha\ (\bar{\lambda} - 0{,}2) + \bar{\lambda}^2\ ]$

buckling curve b $\alpha = 0{,}34$

$\phi = 0{,}5 \cdot [1 + 0{,}34\ (1{,}758 - 0{,}2) + 1{,}758^2\ ] = 2{,}309$

reduction coefficient X = $\frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}}$

X = $\frac{1}{2{,}309 + \sqrt{2{,}309^2 - 1{,}758^2}}$ = 0,263

critical buckling force $ultN_{dD} = 0{,}263 \cdot 40{,}83 \cdot 1{,}8895 = 20{,}26\ kN/m$

decisive design resistance N_{dD} = 20,26 kN/m

- **Step 4**

Interaction bending moment / axial compression

$$\left(\frac{N_D}{N_{dD}}\right)^{0,8} + \left(\frac{M}{M_d}\right)^{0,8} \leq 1$$

$$\left(\frac{18,87}{20,26}\right)^{0,8} + \left(\frac{0,40}{1,093}\right)^{0,8} = 0,945 + 0,447 = 1,39 > 1$$

In this example, the design procedure EN 1993 leads to design on the safe side. Following the procedure with the internal ultimate forces found by test leads to an overflow of 39%. The ultimate internal forces with respect to the design limit 1 are below the failure load from test.

Using buckling curve c is even more safe. Annex page 11 contains the results for all calculated configurations (test families, variation of cross section values, variation of spring stiffness) for buckling curve b as well as for curve c.

6.3 Visualization of the results by graphs, design proposition

The results of the calculations according to chapter 6.1 and 6.2 are shown in the following diagrams.

For the 3 tested setups with the span lengths 3 m, 4 m and 5 m, which represent 3 different slenderness ratios between 1,30 and 2,20, the calculated M-N-interaction under characteristic failure load for the 4 above described design models is indicated. The designation of the graphs is:

- "gross" means, that the internal forces of the arch are calculated using the gross cross section values A_g and J_g of the trapezoidal sheeting
- "effective" means, that the internal forces of the arch are calculated using the effective cross section values A_{ef} and J_{ef} of the trapezoidal sheeting. The effective cross section values are determined for axial compression.
- "DIN" means interaction formula according to DIN 18807 with limitation of $\alpha < 1$ (pure DIN)
- "DIN unltd" means interaction formula according to DIN 18807 without limitation of α (modified DIN)
- "b" means interaction formula according to EN 1993-1-3 using buckling curve b
- "c" means interaction formula according to EN 1993-1-3 using buckling curve c

The M-N-interaction is calculated with the internal forces under characteristic failure load. Results > 1 mean, that the design formula is on the safe side. The maximum load limited by M-N-interaction = 1 would be smaller than the characteristic failure load determined by test. Results < 1 mean, that the design formula is unsafe.

Fig 12 contains the results, if the supports of the arch are considered as fixed in the horizontal sense.

To explain the mode of presentation: There is no test graph as usual in the diagram, which is compared to a theoretical design graph. The internal forces M and N, which represent the test, because they are calculated for characteristic failure load, enter in the chosen interaction formula. If the design model were perfect, the result of the interaction formula would be 1,00. This means, that the internal forces coming from the test would lead to an interaction of 1,0 exactly. If the calculated interaction is greater than 1, the characteristic failure load will be higher than the maximum theoretical M-N-combination which leads per definition to 1,0. The design procedure is safe. If the calculated interaction is below 1, the design procedure will be unsafe: The M-N-combination, which causes failure in test, leads to a calculated interaction below 1, and therefore feigns reserves which don't exist.

If that static model with fixed supports is chosen for the arch, it can be stated:

- The order of the 4 design models regarding the safety is – from unsafe to safe: Pure DIN – DIN without limit of α – EN 1993-1-3 buckling curve b – EN 1993-1-3 buckling curve c

- All 4 design models are unsafe for the test setup with 3 m and with 5 m span. Only for the intermediate span 4 m, the design formulas are more or less safe.

- There is no significant difference on the final result, if the internal forces of the arch are calculated using cross section values of the gross cross section or of the effective cross section.

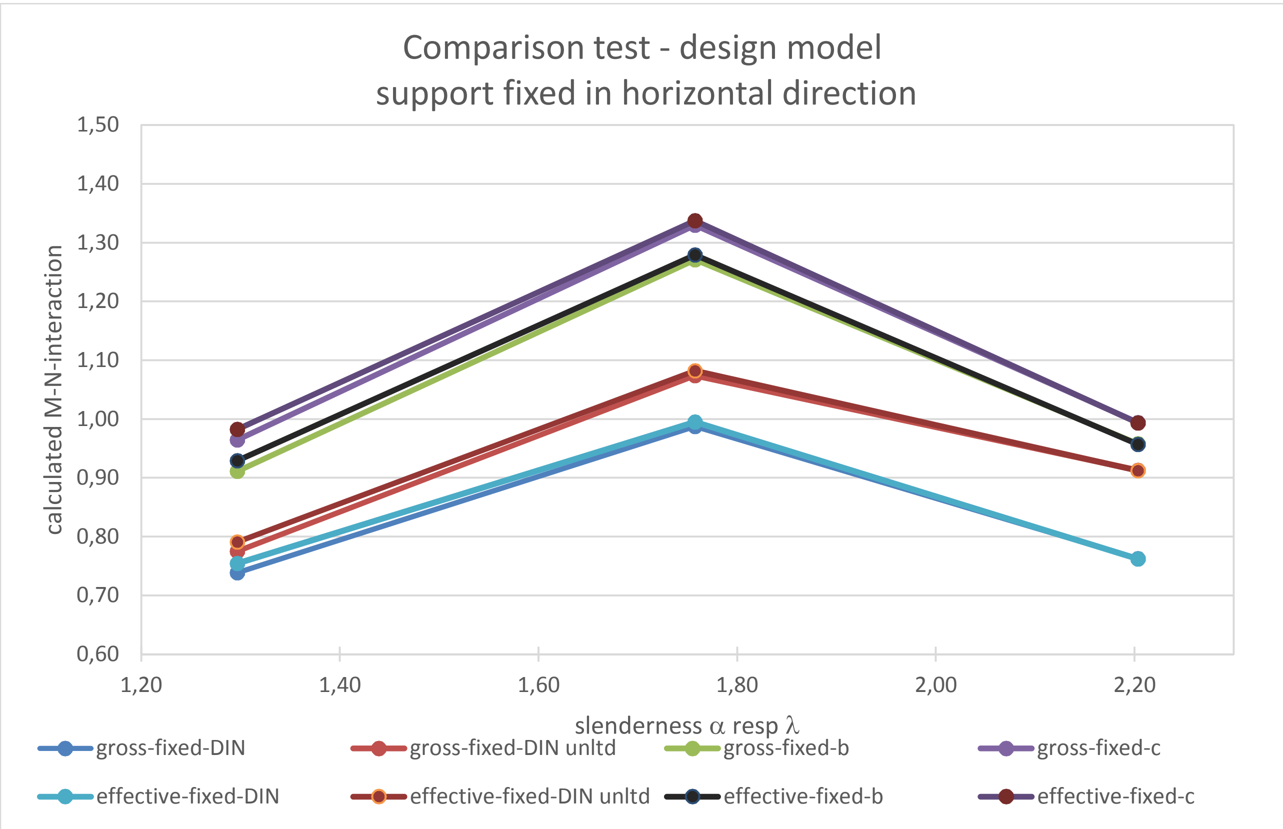

Fig. 12: Design models for M-N-interaction, arch with horizontally fixed supports

Fig 13 contains the results, when the supports of the arch are considered as elastic in the horizontal sense. Depending on the stiffness of the horizontal spring, the deflections of the arch increase, the vertical deflection at summit as well as the horizontal displacement at support. Furthermore the bending moments increase and the axial compression forces decrease; this means, that the "arch effect" decreases. The increase of the bending moments is more important than the decrease of the axial compression forces which leads to more unfavourable results when calculating the interaction formulas. The design models become safer, when the internal forces are calculated with horizontal displacement at support.

When horizontal displacement at support is allowed, a significant difference between the calculation with gross cross section and with effective gross section occurs. Using the gross cross section to calculate the internal forces of the arch leads to a more conservative design.

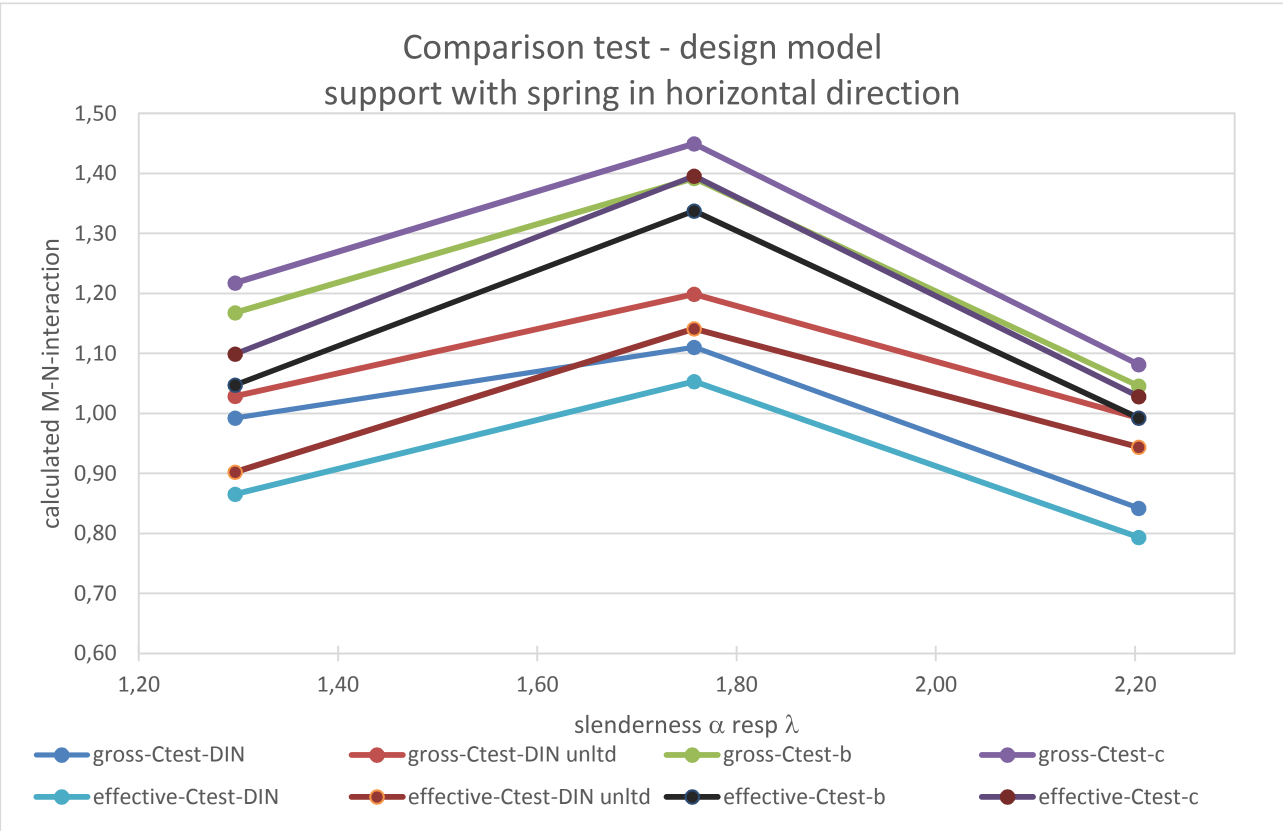

Fig. 13: Design models for M-N-interaction, arch with elastic supports in horizontal sense

Conclusions for a conservative design model for curved profiles with arch effect:

1. The internal forces of the arch (bending moments, axial forces) should be calculated using the gross cross section values A_g and J_g of the profiled sheeting.

2. The horizontal displacement at supports may not be neglected. As grater the displacement is estimated, the internal forces become more unfavourable. Therefore it is necessary to take into account the horizontal displacement by modelling the support with a horizontal spring. The spring stiffness, which depends on the substructure and the fixing of the profiled sheeting, should be adjusted, that the calculated horizontal displacements meet the real values. To avoid unsafe design, the spring stiffness should not be over-estimated. Under-estimation of the spring stiffness leads to an over-estimation of the horizontal displacements and in consequence to a design on the safe side.

3. The bending moment – axial compression – interaction should be calculated with the interaction formula of DIN 18807, but without limitation of α to 1.

4. The design model is verified for arches with symmetric loading. If it is also applicable for arches with not symmetric loading, should be researched in another project.

7. References

[1] Deliverable D 2.3: Test report. Curved profiles. KIT, 31.05.2015

[2] EN 1993-1.3: Eurocode 3 – Design of steel structures. Part 1.3: General rules – supplementary rules for cold-formed members and sheeting

[3] DIN 18807 part 1 and part 3: Trapezoidal sheeting in buildings; steel trapezoidal sheeting.

[4] DIN 18800 part 2: Steel structures – Stability – Buckling of bars and skeletal structures

Annex: Detailed test evaluation